CHAPTER 4

Engineering Simple Interest Rate Derivatives

1. Introduction

Foreign currency and commodity forwards (futures) are the simplest types of derivative instruments. The instruments described in this chapter are somewhat more complicated. The chapter discusses financial engineering methods that use *forward loans*, *Eurocurrency futures*, and *forward rate agreements* (FRAs). The discussion prepares the ground for the next two chapters on swap-based financial engineering. In fact, the FRA contracts considered here are precursors to plain vanilla swaps.

Interest rate strategies, hedging, and risk management present more difficulties than FX, equity, or commodity-related instruments for at least two reasons. First of all, the payoff of an interest rate derivative depends, by definition, on some interest rate(s). In order to price the instrument, one needs to apply discount factors to the future payoffs and calculate the relevant present values. But the discount factor itself is an interest rate-dependent concept. If interest rates are *stochastic*, the present value of an interest rate-dependent cash flow will be a nonlinear random variable; the resulting expectations *may* not be as easy to calculate. There will be *two sources* of any future fluctuations—those due to future cash flows themselves and those due to changes in the *discount factor* applied to these cash flows. When dealing with equity or commodity derivatives, such nonlinearities are either not present or have a relatively minor impact on pricing.

Second, every interest rate is associated with a maturity or *tenor*. This means that, in the case of interest rate derivatives we are not dealing with a single random variable, but with vector-valued stochastic processes. The existence of such a vector-valued random variable requires new methods of pricing, risk management, and strategic position taking.

1.1. A Convergence Trade

Before we start discussing replication of elementary interest rate derivatives we consider a real life example.

For a number of years before the European currency (euro) was born, there was significant uncertainty as to which countries would be permitted to form the group of euro users. During this period, market practitioners put in place the so-called *convergence plays*. The reading that follows is one example.

EXAMPLE:

Last week traders took positions on convergence at the periphery of Europe.

Traders sold the spread between the Italian and Spanish curves. JP Morgan urged its customers to buy a 12×24 Spanish forward rate agreement (FRA) and sell a 12×24 Italian FRA. According to the bank, the spread, which traded at 133 bp would move down to below 50 bp.

The logic of these trades was that if Spain entered the single currency, then Italy would also do so. Recently, the Spanish curve has traded below the Italian curve. According to this logic, the Italian yield curve would converge on the Spanish yield curve, and traders would gain. (Episode based on IFR issue number 1887).

In this episode, traders *buy* and *sell spreads* in order to benefit from a likely occurrence of an event. These spreads are bought and sold using the FRAs, which we discuss in this chapter. If the two currencies converge, the *difference* between Italian and Spanish interest rates will decline.[1] The FRA positions will benefit. Note that market professionals call this *selling* the spread. As the spread goes *down*, they will profit—hence, in a sense they are *short* the spread.

This chapter develops the financial engineering methods that use forward loans, FRAs, and Eurocurrency futures. We first discuss these instruments and obtain contractual equations that can be manipulated usefully to produce other synthetics. The synthetics are used to provide pricing formulas.

2. Libor and Other Benchmarks

We first need to define the concept of Libor rates. The existence of such reliable *benchmarks* is essential for engineering interest rate instruments.

Libor is an arithmetic average interest rate that measures the cost of borrowing from the point of view of a panel of preselected *contributor* banks in London. It stands for London Interbank Offered Rate. It is the ask or offer price of money available only to banks. It is an unsecured rate in the sense that the borrowing bank does not post any collateral. The BBA-Libor is obtained by polling a panel of preselected banks in London.[2] Libor interest rates are published daily at 11:00 London time for nine currencies.

Euribor is a similar concept determined in Brussels by polling a panel of banks in continental Europe. These two benchmarks will obviously be quite similar. London banks and Frankfurt banks face similar risks and similar costs of funding. Hence they will lend euros at approximately the same rate. But Libor and Euribor may have some slight differences due to the composition of the panels used.

Important Libor maturities are overnight, one week, one, two, three, six, nine, and twelve months. A plot of Libor rates against their maturities is called the Libor curve.

Libor is a money market yield and in most currencies it is quoted on the ACT/360 basis. Derivatives written on Libor are called Libor instruments. Using these derivatives and the underlying Euromarket loans, banks create Libor exposure. Tibor (Tokyo) and Hibor (Hong Kong) are examples of other benchmarks that are used for the same purpose.

[1] Although each interest rate may go up or down individually.

[2] BBA stands for the British Bankers Association.

When we use the term "interest rates" in this chapter, we often mean Libor rates. We can now define the major instruments that will be used. The first of these are the forward loans. These are not liquid, but they make a good starting point. We then move to forward rate agreements and to Eurocurrency futures.

3. Forward Loans

A *forward loan* is engineered like any forward contract, except that what is being bought or sold is not a currency or commodity, but instead, a *loan*. At time t_0 we write a contract that will settle at a future date t_1. At settlement the trader receives (delivers) a loan that matures at t_2, $t_1 < t_2$. The contract will specify the interest rate that will apply to this loan. This interest rate is called the *forward rate* and will be denoted by $F(t_0, t_1, t_2)$. The forward rate is determined at t_0. The t_1 is the start date of the future loan, and t_2 is the date at which the loan matures.

The situation is depicted in Figure 4-1. We write a contract at t_0 such that at a future date, t_1, USD100 are received; the principal and interest are paid at t_2. The interest is $F_{t_0}\delta$, where δ is the day-count adjustment, ACT/360:

$$\delta = \frac{t_2 - t_1}{360} \tag{1}$$

To simplify the notation, we abbreviate the $F(t_0, t_1, t_2)$ as F_{t_0}. As in Chapter 3, the day-count convention needs to be adjusted if a year is defined as having 365 days.

Forward loans permit a great deal of flexibility in balance sheet, tax, and risk management. The need for forward loans arises under the following conditions:

- A business would like to *lock in* the "current" *low* borrowing rates from money markets.
- A bank would like to *lock in* the "current" *high* lending rates.
- A business may face a floating-rate liability at time t_1. The business may want to *hedge* this liability by securing a future loan with a known cost.

It is straightforward to see how forward loans help to accomplish these goals. With the forward loan of Figure 4-1, the party has agreed to receive 100 dollars at t_1 and to pay them back at t_2 with interest. The key point is that the interest rate on this forward loan is fixed at time t_0. The forward rate $F(t_0, t_1, t_2)$ "locks in" an unknown future variable at time t_0 and thus eliminates the risk associated with the unknown rate. The L_{t_1} is the Libor interest rate for a $(t_2 - t_1)$ period loan and can be observed only at the future date t_1. Fixing $F(t_0, t_1, t_2)$ will eliminate the risk associated with L_{t_1}.

The chapter discusses several examples involving the use of forward loans and their more recent counterparts, forward rate agreements.

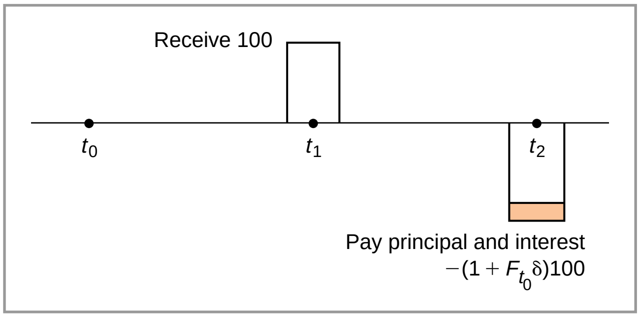

FIGURE 4-1

3.1. *Replication of a Forward Loan*

In this section we apply the techniques developed in Chapter 3 to forward loans and thereby obtain synthetics for this instrument. More than the synthetic itself, we are concerned with the methodology used in creating it. Although forward loans are not liquid and rarely traded in the markets, the synthetic will generate a contractual equation that will be useful for developing contractual equations for FRAs, and the latter *are* liquid instruments.

We begin the engineering of a synthetic forward loan by following the same strategy outlined in Chapter 3. We first decompose the forward loan cash flows into separate diagrams and then try to convert these into known liquid instruments by adding and subtracting appropriate new cash flows. This is done so that, when added together, the extra cash flows cancel each other out and the original instrument is recovered. Figure 4-2 displays the following steps:

1. We begin with the cash flow diagram for the forward loan shown in Figure 4-2a. We *detach* the two cash flows into separate diagrams. Note that at this stage, these cash flows cannot form tradeable contracts. Nobody would want to buy 4-2c, and everybody would want to have 4-2b.

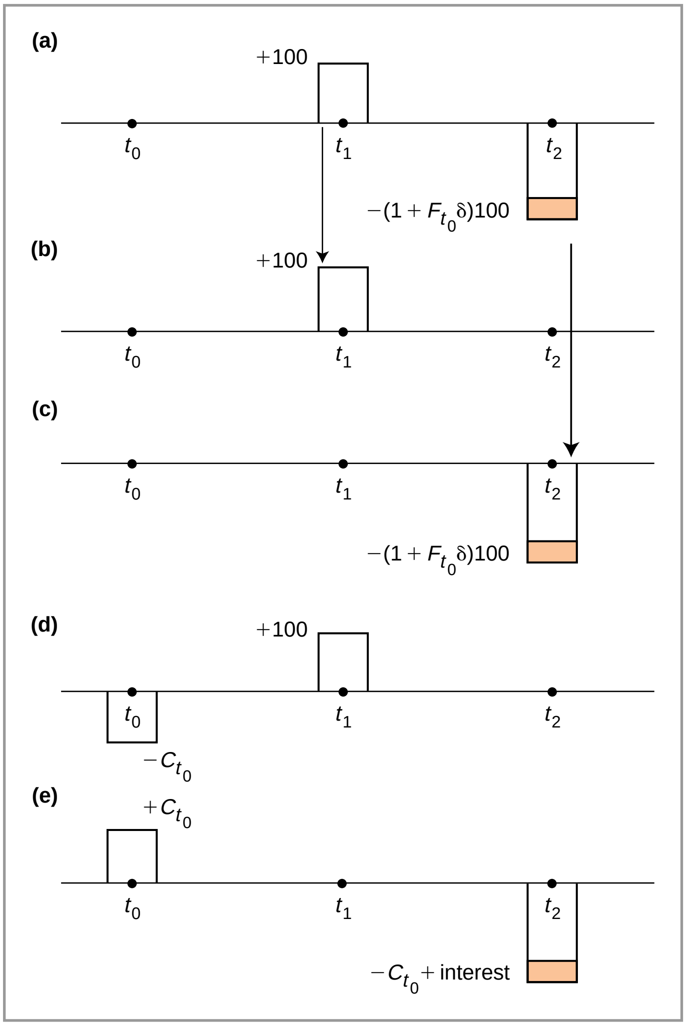

FIGURE 4-2

2. We need to transform these cash flows into tradeable contracts by adding compensating cash flows in each case. In Figure 4-2b we add a *negative* cash flow, preferably at time t_0.[3] This is shown in Figure 4-2d. Denote the size of the cash flow by $-C_{t_0}$.
3. In Figure 4-2c, add a *positive* cash flow at time t_0, to obtain Figure 4-2e. The cash flow has size $+C_{t_0}$.
4. Make sure that the vertical addition of Figures 4-2d and 4-2e will replicate what we started with in Figure 4-2a. For this to be the case, the two newly added cash flows have to be identical in absolute value but different in sign. A vertical addition of Figures 4-2d and 4-2e will cancel any cash exchange at time t_0, and this is exactly what is needed to duplicate Figure 4-2a.[4]

At this point, the cash flows of Figure 4-2d and 4-2e need to be interpreted as specific financial contracts so that the components of the synthetic can be identified. There are many ways to do this. Depending on the interpretation, the synthetic will be constructed using different assets.

3.1.1. Bond Market Replication

As usual, we assume credit risk away. A first synthetic can be obtained using bond and T-bill markets. Although this is not the way preferred by practitioners, we will see that the logic is fundamental to financial engineering. Suppose default-free pure discount bonds of specific maturities denoted by $\{B(t_0, t_i), i = 1, \ldots n\}$ trade actively.[5] They have par value of 100.

Then, within the context of a pure discount bond market, we can interpret the cash flows in Figure 4-2d as a *long* position in the t_1-maturity discount bond. The trader is paying C_{t_0} at time t_0 and receiving 100 at t_1. This means that

$$B(t_0, t_1) = C_{t_0} \tag{2}$$

Hence, the value of C_{t_0} can be determined if the bond price is known.

The synthetic for the forward loan will be fully described once we put a label on the cash flows in Figure 4-2e. What do these cash flows represent? These cash flows look like an appropriate *short* position in a t_2-maturity discount bond.

Does this mean we need to short *one* unit of the $B(t_0, t_2)$? The answer is no, since the time t_0 cash flow in Figure 4-2e has to equal C_{t_0}.[6] However, we know that a t_2-maturity bond will necessarily be cheaper than a t_1-maturity discount bond.

$$B(t_0, t_2) < B(t_0, t_1) = C_{t_0} \tag{3}$$

Thus, shorting *one* t_2-maturity discount bond will not generate sufficient time-t_0 funding for the position in Figure 4-2d. The problem can easily be resolved, however, by shorting not one but λ bonds such that

$$\lambda B(t_0, t_2) = C_{t_0} \tag{4}$$

But we already know that $B(t_0, t_1) = C_{t_0}$. So the λ can be determined easily:

$$\lambda = \frac{B(t_0, t_1)}{B(t_0, t_2)} \tag{5}$$

[3] Otherwise, if we add it at any other time, we get another forward loan.

[4] That is why both cash flows have size C_{t_0} and are of opposite sign.

[5] The $B(t_0, t_i)$ are also called default-free discount factors.

[6] Otherwise, time-t_0 cash flows will not cancel out as we add the cash flows in Figures 4-2d and 4-2e vertically.

According to (3) λ will be greater than one. This particular short position *will* generate enough cash for the long position in the t_1 maturity bond. Thus, we finalized the first synthetic for the forward loan:

$$\{\textit{Buy one } t_1\textit{-discount bond, short } \tfrac{B(t_0,t_1)}{B(t_0,t_2)} \textit{ units of the } t_2\textit{-discount bond}\} \tag{6}$$

To double-check this result, we add Figures 4-2d and 4-2e vertically and recover the original cash flow for the forward loan in Figure 4-2a.

3.1.2. Pricing

If markets are liquid and there are no other transaction costs, arbitrage activity will make sure that the cash flows from the forward loan and from the replicating portfolio (synthetic) are the same. In other words the sizes of the time-t_2 cash flows in Figures 4-2a and 4-2e should be equal. This implies that

$$1 + F(t_0, t_1, t_2)\delta = \frac{B(t_0, t_1)}{B(t_0, t_2)} \tag{7}$$

where the δ is, as usual, the day-count adjustment.

This arbitrage relationship is of fundamental importance in financial engineering. Given liquid bond prices $\{B(t_0, t_1), B(t_0, t_2)\}$, we can price the forward loan *off* the bond markets using this equation. More important, equality (7) shows that there is a crucial relationship between forward rates at different maturities and discount bond prices. But discount bond prices are *discounts* which can be used in obtaining the present values of future cash flows. This means that forward rates are of primary importance in pricing and risk managing financial securities.

Before we consider a second synthetic for the forward loan, we prefer to discuss how all this relates to the notion of arbitrage.

3.1.3. Arbitrage

What happens when the equality in formula (7) breaks down? We analyze two cases assuming that there are no bid-ask spreads. First, suppose market quotes at time t_0 are such that

$$(1 + F_{t_0}\delta) > \frac{B(t_0, t_1)}{B(t_0, t_2)} \tag{8}$$

where the forward rate $F(t_0, t_1, t_2)$ is again abbreviated as F_{t_0}. Under these conditions, a market participant can secure a synthetic forward loan in bond markets at a cost below the return that could be obtained from lending in forward loan markets. This will guarantee positive arbitrage gains. This is the case since the "synthetic" *funding cost*, denoted by $F^*_{t_0}$,

$$F^*_{t_0} = \frac{B(t_0, t_1)}{\delta B(t_0, t_2)} - \frac{1}{\delta} \tag{9}$$

will be less than the forward rate, F_{t_0}. The position will be riskless if it is held until maturity date t_2.

These arbitrage gains can be secured by (1) shorting $\frac{B(t_0,t_1)}{B(t_0,t_2)}$ units of the t_2-bond, which generates $B(t_0, t_1)$ dollars at time t_0, then (2) using these funds buying one t_1-maturity bond, and (3) at time t_1 lending, at rate F_{t_0}, the 100 received from the maturing bond. As a result of these operations, at time t_2, the trader would owe $\frac{B(t_0,t_1)}{B(t_0,t_2)}100$ and would receive $(1 + F_{t_0}\delta)100$. The latter amount is greater, given the condition (8).

Now consider the second case. Suppose time-t_0 markets quote:

$$(1 + F_{t_0}\delta) < \frac{B(t_0, t_1)}{B(t_0, t_2)} \tag{10}$$

Then, one can take the reverse position. Buy $\frac{B(t_0,t_1)}{B(t_0,t_2)}$ units of the t_2-bond at time t_0. To fund this, short a $B(t_0, t_1)$ bond and borrow 100 forward. When time t_2 arrives, receive the $\frac{B(t_0,t_1)}{B(t_0,t_2)}100$ and pay off the forward loan. This strategy can yield arbitrage profits since the funding cost during $[t_1, t_2]$ is lower than the return.

3.1.4. Money Market Replication

Now assume that all maturities of deposits up to 1 year are quoted actively in the interbank money market. Also assume there are no arbitrage opportunities. Figure 4-3 shows how an alternative synthetic can be created. The cash flows of a forward loan are replicated in Figure 4-3a. Figure 4-3c shows a Euromarket loan. C_{t_0} is borrowed at the interbank rate $L^2_{t_0}$.[7] The time-t_2 cash flow in Figure 4-3c needs to be discounted using this rate. This gives

$$C_{t_0} = \frac{100(1 + F_{t_0}\delta)}{(1 + L^2_{t_0}\delta^2)} \tag{11}$$

where $\delta^2 = (t_2 - t_0)/360$.

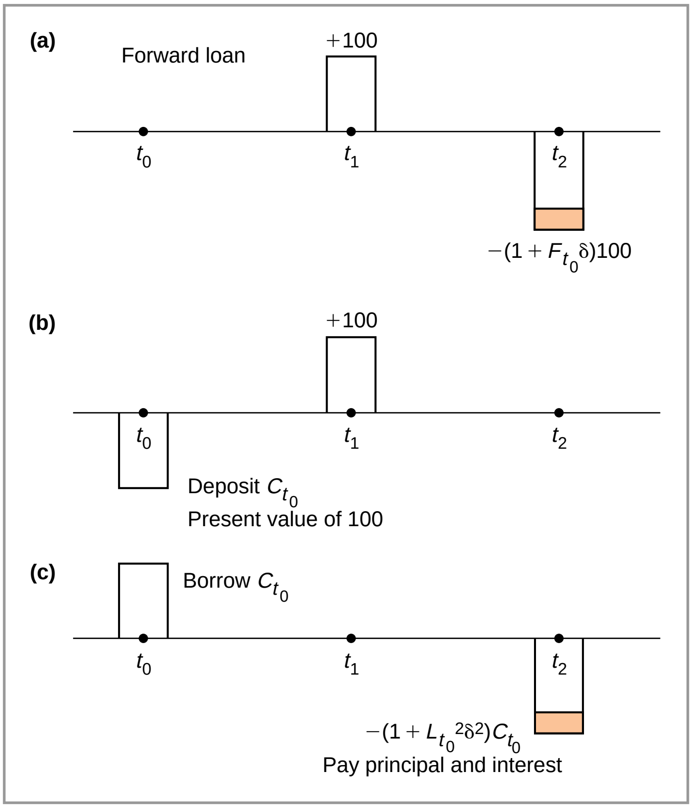

FIGURE 4-3

[7] Here the $L^2_{t_0}$ means the time-t_0 Libor rate for a "cash" loan that matures at time t_2.

Then, C_{t_0} is immediately redeposited at the rate $L^1_{t_0}$ at the shorter maturity. To obtain

$$C_{t_0}(1 + L^1_{t_0}\delta^1) = 100 \tag{12}$$

with $\delta^1 = (t_1 - t_0)/360$. This is shown in Figure 4-3b.

Adding Figures 4-3b and 4-3c vertically, we again recover the cash flows of the forward loan. Thus, the two Eurodeposits form a second synthetic for the forward loan.

3.1.5. Pricing

We can obtain another pricing equation using the money market replication. In Figure 4-3, if the credit risks are the same, the cash flows at time t_2 would be equal, as implied by equation (11). This can be written as

$$(1 + F_{t_0}\delta)100 = C_{t_0}(1 + L^2_{t_0}\delta^2) \tag{13}$$

where $\delta = (t_2 - t_1)/360$. We can substitute further from formula (12) to get the final pricing formula:

$$(1 + F_{t_0}\delta)100 = \frac{100(1 + L^2_{t_0}\delta^2)}{(1 + L^1_{t_0}\delta^1)} \tag{14}$$

Simplifying,

$$(1 + F_{t_0}\delta) = \frac{1 + L^2_{t_0}\delta^2}{1 + L^1_{t_0}\delta^1} \tag{15}$$

This formula prices the forward loan off the money markets. The formula also shows the important role played by *Libor* interest rates in determining the forward rates.

3.2. Contractual Equations

We can turn these results into analytical contractual equations. Using the bond market replication, we obtain

Forward loan that begins at t_1 and ends at t_2	$=$	Short $B(t_0, t_1)/B(t_0, t_2)$ units of t_2 maturity bond	$+$	Long a t_1-maturity bond	(16)

If we use the money markets to construct the synthetic, the contractual equation becomes

Forward loan that begins t_1 and ends at t_2	$=$	Loan with maturity t_2	$+$	Deposit with maturity t_1	(17)

These contractual equations can be exploited for finding solutions to some routine problems encountered in financial markets although they do have drawbacks. Ignoring these for the time being we give some examples.

3.3. Applications

Once a contractual equation for a forward loan is obtained, it can be algebraically manipulated as in Chapter 3, to create further synthetics. We discuss two such applications in this section.

3.3.1. Application 1: Creating a Synthetic Bond

Suppose a trader would like to buy a t_1-maturity bond at time t_0. The trader also wants this bond to be *liquid*. Unfortunately, he discovers that the only bond that is liquid is an *on-the-run* Treasury with a longer maturity of t_2. All other bonds are *off-the-run*.[8] How can the trader create the liquid short-term bond synthetically assuming that all bonds are of discount type and that, contrary to reality, forward loans are liquid?

Rearranging equation (16), we get

Long t_1-maturity bond	=	Forward loan from t_1 to t_2	$-$	Short $B(t_0,t_1)/B(t_0,t_2)$ units of t_2-maturity bond	(18)

The minus sign in front of a contract implies that we need to *reverse* the position. Doing this, we see that a t_1-maturity bond can be constructed synthetically by arranging a forward loan from t_1 to t_2 and then by going *long* $\frac{B(t_0,t_1)}{B(t_0,t_2)}$ units of the bond with maturity t_2. The resulting cash flows would be identical to those of a short bond. More important, if the forward loan and the long bond are liquid, then the synthetic will be more liquid than any existing off-the-run bonds with maturity t_1. This construction is shown in Figure 4-4.

3.3.2. Application 2: Covering a Mismatch

Consider a bank that has a *maturity mismatch* at time t_0. The bank has borrowed t_1-maturity funds from Euromarkets and lent them at maturity t_2. Clearly, the bank has to roll over the short-term loan that becomes due at time t_1 with a new loan covering the period $[t_1, t_2]$. This new loan carries an (unknown) interest rate L_{t_1} and creates a mismatch risk. The contractual equation in formula (17) can be used to determine a *hedge* for this mismatch, by creating a synthetic forward loan, and, in this fashion, locking in time-t_1 funding costs.

In fact, we know from the contractual equation in formula (17) that there is a relationship between short and long maturity loans:

t_2-maturity loan	=	Forward loan from t_1 to t_2	$-$	t_1-maturity deposit	(19)

[8] An on-the-run bond is a liquid bond that is used by traders for a given maturity. It is the latest issue at that maturity. An off-the-run bond has already ceased to have this function and is not liquid. It is kept in investors' portfolios.

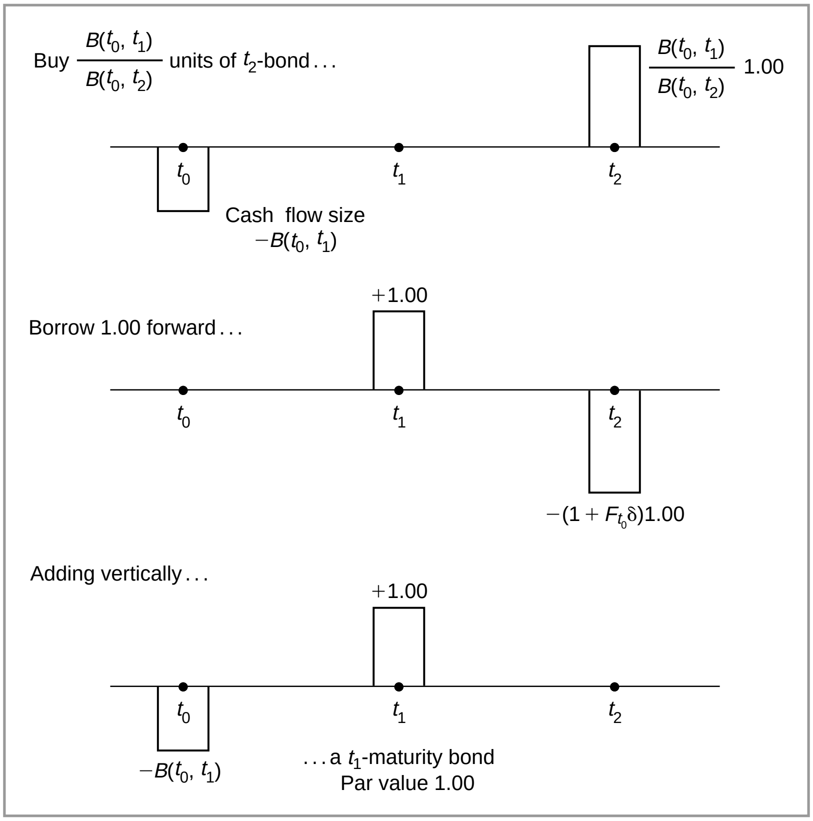

FIGURE 4-4

Changing signs, this becomes

$$\boxed{t_2\text{-maturity loan}} = \boxed{\begin{array}{l}\text{Forward loan from}\\ t_1 \text{ to } t_2\end{array}} + \boxed{t_1\text{-maturity loan}} \tag{20}$$

According to this the forward loan converts the short loan into a longer maturity loan and in this way eliminates the mismatch.

4. Forward Rate Agreements

A forward loan contract implies not one but *two* obligations. First, 100 units of currency will have to be received at time t_1, and second, interest F_{t_0} has to be paid. One can see several drawbacks to such a contract:

1. The forward borrower may not necessarily want to receive cash at time t_1. In most hedging and arbitraging activities, the players are trying to *lock in* an unknown interest rate and are not necessarily in need of "cash." A case in point is the convergence play described

in Section 2, where practitioners were receiving (future) Italian rates and paying (future) Spanish rates. In these strategies, the objective of the players was to *take a position* on Spanish and Italian interest rates. None of the parties involved had any wish to end up with a loan in one or two years.

2. A second drawback is that forward loan contracts involve *credit risk*. It is not a good idea to put a credit risk on a balance sheet if one wanted to lock in an interest rate.[9]
3. These attributes may make speculators and arbitrageurs stay away from any potential forward loan markets, and the contract may be *illiquid*.

These drawbacks make the forward loan contract a less-than-perfect financial engineering instrument. A good instrument would *separate* the credit risk and the interest rate commitment that coexist in the forward loan. It turns out that there is a nice way this can be done.

4.1. Eliminating the Credit Risk

First, note that a player using the forward loan *only* as a tool to lock in the future Libor rate L_{t_1} will immediately have to relend the USD100 received at time t_1 at the going market rate L_{t_1}. Figure 4-5a displays a forward loan committed at time t_0. Figure 4-5b shows the corresponding

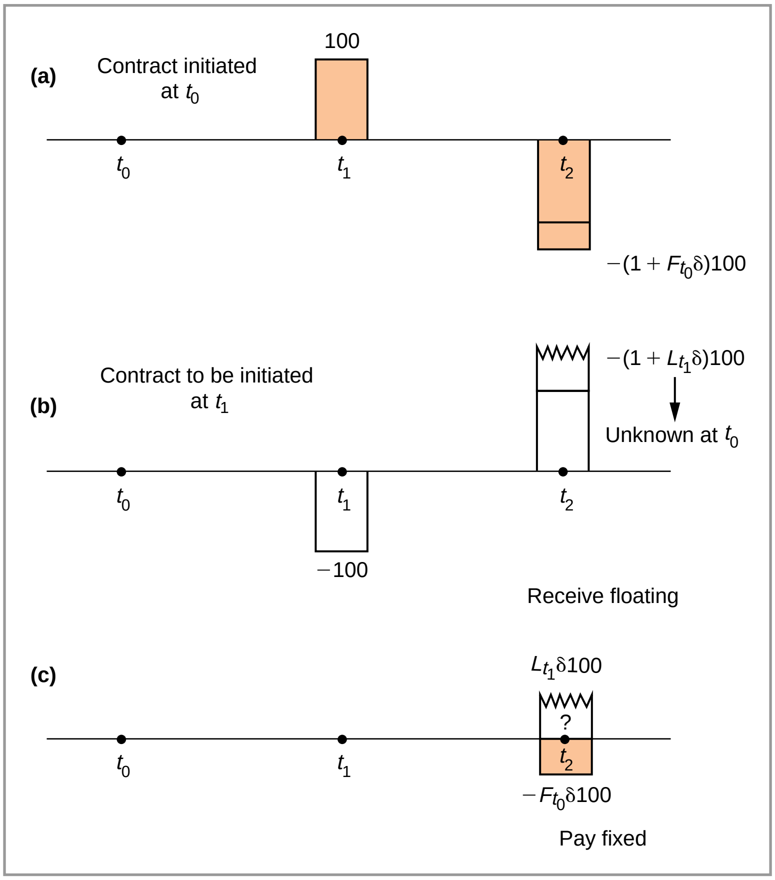

FIGURE 4-5

[9] Note that the forward loan in Figure 4-1 assumes the credit risk away.

spot deposit. The practitioner waits until time t_1 and then makes a deposit at the rate L_{t_1}, which will be known at that time. This "swap" *cancels* an obligation to receive 100 and ends up with only the fixed rate F_{t_0} commitment.

Thus, the joint use of a forward loan, and a spot deposit *to be* made in the future, is sufficient to reach the desired objective—namely, to eliminate the risk associated with the unknown Libor rate L_{t_1}. These steps will lock in F_{t_0}. We consider the result of this strategy in Figure 4-5c. Add vertically the cash flows of the forward loan (4-5a) and the spot loan (4-5b). Time-t_1 cash flows cancel out since they are in the same currency. Time-t_2 payment and receipt of the principal will also cancel. What is left is the respective interest payments. This means that the portfolio consisting of

$$\{A \text{ forward loan for } t_1 \text{ initiated at } t_0, \text{ a spot deposit at } t_1\} \tag{21}$$

will lead, according to Figure 4-5c, to the following (net) cash flows:

	Cash paid	**Cash received**	**Total**
Time t_1	-100	$+100$	0
Time t_2	$-100(1+F_{t_0}\delta)$	$100(1+L_{t_1}\delta)$	$100(L_{t_1}-F_{t_0})\delta$

Thus, letting the principal of the forward loan be denoted by the parameter N, we see that the portfolio in expression (21) results in a time-t_2 net cash flow equaling

$$N(L_{t_1} - F_{t_0})\delta \tag{22}$$

where δ is the day's adjustment to interest, as usual.

4.2. Definition of the FRA

This is exactly where the FRA contract comes in. If a client has the objective of locking in the future borrowing or lending *costs* using the portfolio in (21), why not offer this to him or her in a *single* contract? This contract will involve *only* the exchange of two interest payments shown in Figure 4-5c.

In other words, we write a contract that specifies a notional amount, N, the dates t_1 and t_2, and the "price" F_{t_0}, with payoff $N(L_{t_1} - F_{t_0})\delta$.[10] This instrument is a *paid-in-arrears* forward rate agreement or a FRA.[11] In a FRA contract, the *purchaser* accepts the receipt of the following sum at time t_2:

$$(L_{t_1} - F_{t_0})\delta N \tag{23}$$

if $L_{t_1} > F_{t_0}$ at date t_1. On the other hand, the purchaser pays

$$(F_{t_0} - L_{t_1})\delta N \tag{24}$$

if $L_{t_1} < F_{t_0}$ at date t_1. Thus, the buyer of the FRA will pay *fixed* and *receive* floating.

[10] The N represents a *notional* principal since the principal amount will never be exchanged. However, it needs to be specified in order to determine the amount of interest to be exchanged.

[11] It is paid-in-arrears because the unknown interest, L_{t_1}, will be known at time t_1, the interest payments are exchanged at time t_2, when the forward (fictitious) loan is due.

In the case of market-traded FRA contracts, there is one additional complication. The settlement is *not* done in-arrears at time t_2. Instead, FRAs are settled at time t_1, and the transaction will involve the following discounted cash flows. The

$$\frac{(L_{t_1} - F_{t_0})\delta N}{1 + L_{t_1}\delta} \tag{25}$$

will be received at time t_1, if $L_{t_1} > F_{t_0}$ at date t_1. On the other hand,

$$\frac{(F_{t_0} - L_{t_1})\delta N}{1 + L_{t_1}\delta} \tag{26}$$

will be paid at time t_1, if $L_{t_1} < F_{t_0}$. Settling at t_1 instead of t_2 has one subtle advantage for the FRA seller, which is often a bank. If during $[t_0, t_1]$ the interest rate has moved in favor of the bank, time-t_1 settlement will reduce the marginal credit risk associated with the payoff. The bank can then operate with a lower credit line.

4.2.1. An Interpretation

Note one important interpretation. A FRA contract can be visualized as an *exchange* of two interest payments. The purchaser of the FRA will be paying the known interest $F_{t_0}\delta N$ and is accepting the (unknown) amount $L_{t_1}\delta N$. Depending on which one is greater, the settlement will be a receipt or a payment. The sum $F_{t_0}\delta N$ can be considered, as of time t_0, as the fair payment market participants are willing to make against the random and unknown $L_{t_1}\delta N$. It can be regarded as the time to "market value" of $L_{t_1}\delta N$.

4.3. *FRA Contractual Equation*

We can immediately obtain a synthetic FRA using the ideas displayed in Figure 4-5. Figure 4-5 displays a swap of a fixed rate loan of size N, against a floating rate loan of the same size. Thus, we can write the contractual equation

Buying a FRA	=	Fixed rate loan starting t_1 ending t_2	+	Floating rate deposit starting t_1 ending t_2	(27)

It is clear from the construction in Figure 4-5 that the FRA contract eliminates the credit risk associated with the principals—since the two N's will cancel out—but leaves behind the exchange of interest rate risk. In fact, we can push this construction further by "plugging in" the contractual equation for the fixed rate forward loan obtained in formula (17) and get

Buying a FRA	=	Loan with maturity t_2	+	Deposit with maturity t_1	+	Spot deposit starting t_1 ending t_2

(28)

This contractual equation can then be exploited to create new synthetics. One example is the use of FRA strips.

4.3.1. Application: FRA Strips

Practitioners use portfolios of FRA contracts to form *FRA strips*. These in turn can be used to construct synthetic loans and deposits and help to hedge swap positions. The best way to understand FRA strips is with an example that is based on the contractual equation for FRAs obtained earlier.

Suppose a market practitioner wants to replicate a 9-month fixed-rate borrowing synthetically. Then the preceding contractual equation implies that the practitioner should take a cash loan at time t_0, pay the Libor rate L_{t_0}, *and* buy a *FRA strip* made of *two* sequential FRA contracts, a (3×6) FRA and a (6×9) FRA. This will give a synthetic 9-month fixed-rate loan. Here the symbol (3×6) means t_2 is 6 months and t_1 is 3 months.

5. Futures: Eurocurrency Contracts

Forward loans do not trade in the OTC market because FRAs are much more cost-effective. Eurocurrency futures are another attractive alternative. In this section, we discuss Eurocurrency futures using the Eurodollar (ED) futures as an example and then compare it with FRA contracts. This comparison illustrates some interesting aspects of successful contract design in finance.

FRA contracts involve exchanges of interest payments associated with a floating and a fixed-rate loan. The Eurodollar futures contracts trade future loans *indirectly*. The settlement will be in cash and the futures contract will again result only in an exchange of interest rate payments. However, there are some differences with the FRA contracts.

Eurocurrency futures trade the forward loans (deposits) shown in Figure 4-1 as homogenized contracts. These contracts deal with loans and deposits in *Euromarkets*, as suggested by their name. The *buyer* of the Eurodollar futures contract is a potential *depositor* of 3-month Eurodollars and will lock in a future deposit rate.

Eurocurrency futures contracts do not deliver the deposit itself. At expiration date t_1, the contract is cash settled. Suppose we denote the price of the futures contract quoted in the market by Q_{t_0}. Then the *buyer* of a 3-month Eurodollar contract "promises" to deposit $100(1 - \tilde{F}_{t_0}\frac{1}{4})$ dollars at expiration date t_1 and receive 100 in 3 months. The *implied* annual interest rate on this loan is then calculated by the formula

$$\tilde{F}_{t_0} = \frac{100.00 - Q_{t_0}}{100} \tag{29}$$

This means that the price quotations are related to forward rates through the formula

$$Q_{t_0} = 100.00(1 - \tilde{F}_{t_0}) \tag{30}$$

However, there are important differences with forward loans. The interest rate convention used for forward loans is equivalent to a *money market yield*. For example, to calculate the time-t_1 present value at time t_0 we let

$$PV(t_0, t_1, t_2) = \frac{100}{(1 + F_{t_0}\delta)} \tag{31}$$

Futures contracts, on the other hand, use a convention similar to *discount rates* to calculate the time-t_1 value of the forward loan

$$\tilde{PV}(t_0, t_1, t_2) = 100(1 - \tilde{F}_{t_0}\delta) \tag{32}$$

If we want the amount traded to be the same:

$$PV(t_0, t_1, t_2) = \tilde{PV}(t_0, t_1, t_2), \tag{33}$$

the two forward rates on the right-hand side of formulas (31) and (32) *cannot* be identical. Of course, there are many other reasons for the right-hand side and left-hand side in formula (33) not to be the same. Futures markets have mark-to-market; FRA markets, in general, do not. With mark-to-market, gains and losses occur daily, and these daily cash flows may be *correlated* with the overnight funding rate. Thus, the forward rates obtained from FRA markets need to be adjusted to get the forward rate in the Eurodollar futures, and vice versa.

EXAMPLE:

Suppose at time t_0, futures markets quote a price

$$Q_{t_0} = 94.67 \tag{34}$$

for a Eurodollar contract that expires on the third Wednesday of December 2002. This would mean two things. First, the implied forward rate for that period is given by:

$$F_{t_0} = \frac{100.00 - 94.67}{100} = 0.0533 \tag{35}$$

Second, the contract involves a position on the delivery of

$$100\left(1 - .0533\frac{1}{4}\right) = 98.67 \tag{36}$$

dollars on the third Wednesday of December 2002.

At expiry these funds will never be deposited explicitly. Instead, the contract will be cash settled. For example, if on expiration the exchange has set the delivery settlement price at $Q_{t_1} = 95.60$, this would imply a forward rate

$$F_{t_1} = \frac{100 - 95.60}{100} = 0.0440 \tag{37}$$

and a settlement

$$100\left(1 - .0440\frac{1}{4}\right) = 98.90 \tag{38}$$

Thus, the buyer of the original contract will be compensated as if he or she is making a deposit of 98.67 and receiving a loan of 98.90. The net gain is

$$98.90 - 98.67 = 0.23 \qquad \textit{per 100 dollars} \tag{39}$$

This gain can be explained as follows. When the original position was taken, the (forward) rate for the future 3-month deposit was 5.33%. Then at settlement this rate declined to 4.4%.

Actually, the above example is a simplification of reality as the gains would never be received as a lump-sum at the expiry due to marking-to-market. The mark-to-market adjustments would lead to a gradual accumulation of this sum in the buyer's account. The gains will earn some interest as well. This creates another complication. Mark-to-market gains losses may be correlated with daily interest rate movements applied to these gains (losses).

5.1. Other Parameters

There are some other important parameters of futures contracts. Instead of discussing these in detail, we prefer to report contract descriptions directly. The following table describes this for the CME Eurodollar contract.

Delivery months	: March, June, September, December (for 10 years)
Delivery (Expiry) day	: Third Wednesday of delivery month
Last trading day	: 11.00 Two business days before expiration
Minimum tick	: 0.0025 (for spot-month contract)
"Tick value"	: USD 6.25
Settlement rule	: BBA Libor on the settlement date

The design and the conventions adopted in the Eurodollar contract may seem a bit odd to the reader, but the contract is a successful one. First of all, quoting Q_{t_0} instead of the forward rate $\tilde{F}_{t_0}$ makes the contract similar to buying and selling a futures contract on T-bills. This simplifies related hedging and arbitrage strageties. Second, as mentioned earlier, the contract is settled in cash. This way, the functions of securing a loan and locking in an interest rate are successfully separated.

Third, the convention of using a *linear* formula to represent the relationship between Q_{t_0} and $\tilde{F}_{t_0}$ is also a point to note. Suppose the underlying time-t_1 deposit is defined by the following equation

$$D(t_0, t_1, t_2) = 100(1 - \tilde{F}_{t_0}\delta) \tag{40}$$

A small variation of the forward rate $\tilde{F}_{t_0}$ will result in a *constant* variation in $D(t_0, t_1, t_2)$:

$$\frac{\partial D(t_0, t_1, t_2)}{\partial \tilde{F}_{t_0}} = -\delta 100 = -25 \tag{41}$$

Thus, the *sensitivity* of the position with respect to the underlying interest rate risk is constant, and the product is truly *linear* with respect to $\tilde{F}_{t_0}$.

5.1.1. The "TED Spread"

The difference between the interest rates on Treasury Notes (T-Notes) and Eurodollar (ED) futures is called the *TED spread*. T-Note rates provide a measure of the U.S. government's medium term borrowing costs. Eurodollar futures relate to short-term private sector borrowing costs. Thus the "TED spread" has credit risk elements in it.[12]

Traders form strips of Eurodollar futures and trade them against T-Notes of similar maturity. A similar spread can be put together using Treasury Bills (T-bills) and Eurodollars as well. Given the different ways of quoting yields, calculation of the spread involves some technical adjustments. T-Notes use bond equivalent yields whereas Eurodollars are quoted similar to discount rate basis. The calculation of the TED spread requires putting together strips of futures while adjusting for these differences. There are several technical points that arise along the way.

Once the TED spread is calculated, traders put on trades to benefit from changes in the yield curve slope and in private sector credit risk. For example, traders would *long* the TED spread if

[12] During the credit crisis of 2007–2008, TED spread was often used as a measure of banking sector credit risk.

they expected the yield spread to *widen*. In the opposite case, they would *short* the TED spread and would thus benefit from the *narrowing* of the yield spread.

5.2. *Comparing FRAs and Eurodollar Futures*

A brief comparison of FRAs with Eurocurrency futures may be useful. (1) Being OTC contracts, FRAs are more flexible instruments, since Eurodollar futures trade in terms of preset *homogeneous* contracts. (2) FRAs have the advantage of *confidentiality*. There is no requirement that the FRA terms be announced. The terms of a Eurocurrency contract are known. (3) There are, in general, no *margin requirements* for FRAs and the *mark-to-market requirements* are less strict. With FRAs, money changes hands only at the settlement date. Eurocurrency futures come with margin requirements as well as with mark-to-market requirements. (4) FRAs have *counterparty risk*, whereas the credit risk of Eurocurrency futures contracts are insignificant. (5) FRAs are quoted on an interest rate basis while Eurodollar futures are quoted on a price basis. Thus a trader who sells a FRA will hedge this position by selling a Eurodollar contract as well. (6) Finally, an interesting difference occurs with respect to *fungibility*. Eurocurrency contracts are fungible, in the sense that contracts with the same expiration can be netted against each other even if they are entered into at different times and for different purposes. FRA contracts cannot be netted against each other even with respect to the same counterparty, unless the two sides have a specific agreement.

5.2.1. Convexity Differences

Besides these structural differences, FRAs and Eurocurrency futures have different convexities. The pricing equation for Eurocurrency futures is linear in $\tilde{F}_{t_0}$, whereas the market traded FRAs have a pricing equation that is nonlinear in the corresponding Libor rate. We will see that this requires *convexity adjustments*, which is one reason why we used different symbols to denote the two forward rates.

5.3. *Hedging FRAs with Eurocurrency Futures*

For short-dated contracts, convexity and other differences may be negligible, and we may ask the following question. Putting convexity differences aside, can we hedge a FRA position with futures, and vice versa?

It is best to answer this question using an example. The example also illustrates some real-world complications associated with this hedge.

EXAMPLE:

Suppose we are given the following Eurodollar futures prices on June 17, 2002:

September price (delivery date: September 16) 96.500 (implied rate = 3.500)

December price (delivery date: December 16) 96.250 (implied rate = 3.750)

March price (delivery date: March 17) 96.000 (implied rate = 4.000)

A trader would like to sell a *(3 × 6) FRA on June 17, with a notional amount of USD100,000,000. How can the deal be hedged using these futures contracts?*

Note first that according to the value and settlement date conventions, the FRA will run for the period September 19 through December 19 and will encompass 92 days. It will settle against the Libor fixed on September 17. The September futures contract, on the *other hand, will settle against the Libor fixed on September 16 and is quoted on a 30/360 basis. Thus, the implied forward rates will not be identical for this reason as well.*

Let f be the FRA rate and ϵ be the differences between this rate and the forward rate implied by the futures contract. Using formula (25), the FRA settlement, with notional value of 100 million USD, may be written as

$$\frac{100m\left((0.035+\epsilon)-Libor\right)\frac{92}{360}}{\left(1+Libor\frac{92}{360}\right)} \tag{42}$$

Note that this settlement is discounted to September 19 and will be received once the relevant Libor rate becomes known. Ignoring mark-to-market and other effects, a futures contract covering similar periods will settle at

$$\alpha\left(1m(0.0350-Libor)\frac{90}{360}\right) \tag{43}$$

Note at least two differences. First, the contract has a nominal value of USD1 million. Second, 1 month is, by convention, taken as 30 days, while in the case of FRA it was the actual number of days. The α is the number of contracts that has to be chosen so that the FRA position is correctly hedged.

The trader has to choose α such that the two settlement amounts are as close as possible. This way, by taking opposite positions in these contracts, the trader will hedge his or her risks.

5.3.1. Some Technical Points

The process of hedging is an approximation that may face several technical and practical difficulties. To illustrate them we look at the preceding example once again.

1. Suppose we tried to hedge (or price) a *strip* of FRAs rather than having a single FRA be adjusted to contract using a strip of available futures contracts. Then the strip of FRAs will have to deal with increasing notional amounts. Given that futures contracts have *fixed* notional amounts, contract numbers need to be adjusted instead.
2. As indicated, a 3-month period in futures markets is 90 days, whereas FRA contracts count the actual number of days in the corresponding 3-month period.
3. Given the convexity differences in the pricing formulas, the forward rates implied by the two contracts are not the same and, depending on Libor volatility, the difference may be large or small.
4. There may be differences of 1 or 2 days in the fixing of the Libor rates in the two contracts.

These technical differences relate to this particular example, but they are indicative of most hedging and pricing activity.

6. Real-World Complications

Up to this point, the discussion ignored some real-life complications. We made the following simplifications. (1) We ignored bid-ask spreads. (2) Credit risk was assumed away. (3) We ignored the fact that the fixing date in an FRA is, in general, different from the settlement date. In fact this is another date involved in the FRA contract. Let us now discuss these issues.

6.1. Bid-Ask Spreads

We begin with bid-ask spreads. The issue will be illustrated using a bond market construction. When we replicate a forward loan via the bond market, we buy a $B(t_0, t_1)$ bond and short-sell a $B(t_0, t_2)$ bond. Thus, we have to use ask prices for $B(t_0, t_1)$ and bid prices for $B(t_0, t_2)$. This means that the asking price for a forward interest rate will be

$$1 + F_{t_0}^{\text{ask}}\delta = \frac{B(t_0, t_1)^{\text{ask}}}{B(t_0, t_2)^{\text{bid}}} \tag{44}$$

Similarly, when the client sells a FRA, he or she has to use the bid price of the dealers and brokers. Again, going through the bond markets we can get

$$1 + F_{t_0}^{\text{bid}}\delta = \frac{B(t_0, t_1)^{\text{bid}}}{B(t_0, t_2)^{\text{ask}}} \tag{45}$$

This means that

$$F_{t_0}^{\text{bid}} < F_{t_0}^{\text{ask}} \tag{46}$$

The same bid-ask spread can also be created from the money market synthetic using the bid-ask spreads in the money markets

$$1 + F_{t_0}^{\text{ask}}\delta = \frac{1 + L_{t_0}^{1\text{bid}}\delta^1}{1 + L_{t_0}^{2\text{ask}}\delta^2} \tag{47}$$

Clearly, we again have

$$F_{t_0}^{\text{bid}} < F_{t_0}^{\text{ask}} \tag{48}$$

Thus, pricing will normally yield two-way prices.

In market practice, FRA bid-ask spreads are not obtained in the manner shown here. The bid-ask quotes on the FRA rate are calculated by first obtaining a rate from the corresponding Libors and then adding a spread to both sides of it. Many practitioners also use the more liquid Eurocurrency futures to “make” markets.

6.2. An Asymmetry

There is another aspect to using FRAs for hedging purposes. The net return and net cost from an interest rate position will be asymmetric since, whether you buy (pay fixed) or sell (receive fixed), a FRA *always* settles against Libor. But Libor is an offer (asking) rate, and this introduces an asymmetry.

We begin with a hedging of floating borrowing costs. When a company hedges a floating *borrowing* cost, both interest rates from the cash and the hedge will be Libor based. This means that:

- The company pays Libor + margin to the bank that it borrows funds from.
- The company pays the fixed FRA rate to the FRA counterparty for hedging this floating cost.
- Against which the company receives Libor from the FRA counterparty.

Adding all receipts and payments, the net borrowing cost becomes *FRA rate + margin.*

Now consider what happens when a company hedges, say, a 3-month floating *receipt*. The relevant rate for the cash position is Libid, the bid rate for placing funds with the Euromarkets.

But a FRA always settles always against Libor. So the picture will change to

- Company receives Libid, assuming a zero margin.
- Company receives FRA rate.
- Company pays Libor.

Thus, the net return to the company will become FRA-(Libor-Libid).

7. Forward Rates and Term Structure

A detailed framework for fixed income engineering will be discussed in Chapter 15. However, some preliminary modeling of the term structure is in order. This will clarify the notation and some of the essential concepts.

7.1. Bond Prices

Let $\{B(t_0, t_i), i = 1, 2 \ldots, n\}$ represent the *bond price family*, where each $B(t_0, t_i)$ is the price of a default-free zero-coupon bond that matures and pays \$1 at time t_i. These $\{B(t_0, t_i)\}$ can also be viewed as a vector of *discounts* that can be used to value default-free cash flows.

For example, given a complicated default-free asset, A_{t_0}, that pays deterministic cash flows $\{C_{t_i}\}$ occurring at arbitrary times, $t_i, i = 1, \ldots, k$, we can obtain the value of the asset easily if we assume the following bond price process:

$$A_{t_0} = \sum_i C_{t_i} B(t_0, t_i) \tag{49}$$

That is to say, we just multiply the t_ith cash flow with the current value of one unit of currency that belongs to t_i, and then sum over i.

This idea has an immediate application in the pricing of a coupon bond. Given a coupon bond with a nominal value of \$1 that pays a coupon rate of $c\%$ at times t_i, the value of the bond can easily be obtained using the preceding formula, where the last cash flow will include the principal as well.

7.2. What Forward Rates Imply

In this chapter, we obtained the important arbitrage equality

$$1 + F(t_0, t_1, t_2)\delta = \frac{B(t_0, t_1)}{B(t_0, t_2)} \tag{50}$$

where the $F(t_0, t_1, t_2)$ is written in the expanded form to avoid potential confusion.[13] It implies a forward rate that applies to a loan starting at t_1 and ending at t_2. Writing this arbitrage relationship for *all* the bonds in the family $\{B(t_0, t_i)\}$, we see that

$$1 + F(t_0, t_0, t_1)\delta = \frac{B(t_0, t_0)}{B(t_0, t_1)} \tag{51}$$

[13] Here the δ has no i subscript. This means that the periods $t_i - t_{i-1}$ are constant across i and are given by $(t_i - t_{i-1})/360$.

$$1 + F(t_0, t_1, t_2)\delta = \frac{B(t_0, t_1)}{B(t_0, t_2)} \tag{52}$$

$$\ldots\ldots \tag{53}$$

$$1 + F(t_0, t_{n-1}, t_n)\delta = \frac{B(t_0, t_{n-1})}{B(t_0, t_n)} \tag{54}$$

Successively substituting the numerator on the right-hand side using the previous equality and noting that for the first bond we have $B(t_0, t_0) = 1$, we obtain

$$B(t_0, t_n) = \frac{1}{(1 + F(t_0, t_0, t_1)\delta) \ldots (1 + F(t_0, t_{n-1}, t_n)\delta)} \tag{55}$$

We have obtained an important result. The bond price family $\{B(t_0, t_i)\}$ can be expressed using the forward rate family,

$$\{F(t_0, t_0, t_1), \ldots, F(t_0, t_{n-1}, t_n)\} \tag{56}$$

Therefore if all bond prices are given we can determine the forward rates.

7.2.1. Remark

Note that the "first" forward rate $F(t_0, t_0, t_1)$ is contracted at time t_0 and applies to a loan that starts at time t_0. Hence, it is also the t_0 spot rate:

$$(1 + F(t_0, t_0, t_1)\delta) = (1 + L_{t_0}\delta) = \frac{1}{B(t_0, t_1)} \tag{57}$$

We can write this as

$$B(t_0, t_1) = \frac{1}{(1 + L_{t_0}\delta)} \tag{58}$$

The bond price family $B(t_0, t_i)$ is the relevant *discounts* factors that market practitioners use in obtaining the present values of default-free cash flows. We see that modeling F_{t_0}'s will be quite helpful in describing the modeling of the yield curve or, for that matter, the discount curve.

8. Conventions

FRAs are quoted as two-way prices in bid-ask format, similar to Eurodeposit rates. A typical market contributor will quote a 3-month and a 6-month series.

EXAMPLE:

The 3-month series will look like this:

1×4	4.87	4.91
2×5	4.89	4.94
3×6	4.90	4.95
etc.		

The first row implies that the interest rates are for a 3-month period that will start in 1 month. The second row gives the forward rate for a loan that begins in 2 months for a period of 3 months and so on.

The 6-month series will look like this:

1×7	4.87	4.91
2×8	4.89	4.94
3×9	4.90	4.95
etc.		

According to this table, if a client would like to lock in a fixed payer rate in 3 months for a period of 6 months and for a notional amount of USD1 million, he or she would buy *the* 3s against 9s *and pay the 4.95% rate. For 6 months, the actual net payment of the FRA will be*

$$\frac{1{,}000{,}000\left(\frac{L_{t_3}}{100} - .0495\right)\frac{1}{2}}{\left(1 + \frac{1}{2}\frac{L_{t_3}}{100}\right)} \tag{59}$$

where L_{t_3} is the 6-month Libor rate that will be observed in 3 months.

Another convention is the use of *Libor* rate as a *reference rate* for both the sellers and the buyers of the FRA. *Libor* being an asking rate, one might think that a client who sells a FRA may receive a lower rate than *Libor*. But this is not true, as the reference rate does not change.

9. A Digression: Strips

Before finishing this chapter we discuss an instrument that is the closest real life equivalent to the default-free pure discount bonds $B(t_0, t_i)$. This instrument is called *strips*. U.S. strips have been available since 1985 and UK strips since 1997.

Consider a long-term *straight* Treasury bond, a German bund, or a British gilt and suppose there are no implicit options. These bonds make *coupon* payments during their life at regular intervals. Their day-count and coupon payment intervals are somewhat different, but in essence they are standard long-term debt obligations. In particular, they are not the zero-coupon bonds that we have been discussing in this chapter.

Strips are obtained from coupon bonds. The market practitioner buys a long-term coupon bond and then "strips" each coupon interest payment and the principal and trades them *separately*. Such bonds will be equivalent to zero-coupon bonds except that, if needed, one can put them back together and reconstruct the original coupon bond.

The institution overseeing the government bond market, the Bank of England in the United Kingdom or the Treasury in the United States, arranges the necessary infrastructure to make stripping possible and also designates the strippable securities.[14] Note that only some particular dealers are usually allowed to strip and to reconstruct the underlying bonds. These dealers put in a request to strip a bond that they already have in their account and then they sell the pieces

[14] Stripping a Gilt costs less than $2 and is done in a matter of minutes at the touch of a button. Although it changes depending on the market environment, about 40% of a bond issue is stripped in the United States and in the United Kingdom.

separately.[15] As an example, a 10-year gilt is strippable into 20 coupons plus the principal. There will be 21 zero-coupon bonds with maturities 6, 12, 18, 24 (and so on) months.

10. Conclusions

This chapter has shown, using simple examples, financial engineering applications that use forward loans and FRAs. We obtained new contractual equations and introduced the forward rate (Libor) processes. The chapter continued to build on the simple graphical financial engineering methods that are based on cash flow manipulations.

Suggested Reading

There are many more fixed income instruments involving more complicated parameters than those discussed here. Some of these will certainly be examined in later chapters. But reading some market-oriented books that deal with technical aspects of these instruments may be helpful at this point. Two such books are ***Questa*** *(1999) and* ***Tuckman*** *(2002).* ***Flavell*** *(2002) is another introduction.*

[15] The reason for designing some bonds as strippable is because (1) large bond issues need to be designated and (2) the coupon payment dates need to be such that they fall on the same date, so that when one strips a 2- and a 4-year bond, the coupon strips for the first 2 years become interchangeable. This will increase the liquidity of the strips and also make their maturity more homogeneous.

Exercises

1. You have purchased 1 Eurodollar contract at a price of $Q_0 = 94.13$, with an initial margin of 5%. You keep the contract for 5 days and then sell it by taking the opposite position. In the meantime, you observe the following settlement prices:

$$\{Q_1 = 94.23, Q_2 = 94.03, Q_3 = 93.93, Q_4 = 93.43, Q_5 = 93.53\} \tag{60}$$

   (a) Calculate the string of mark-to-market losses or gains.
   (b) Suppose the spot interest rate during this 5-day period was unchanged at 6.9%. What is the total interest gained or paid on the clearing firm account?
   (c) What are the total gains and losses at settlement?

2. The treasurer of a small bank has borrowed funds for 3 months at an interest rate of 6.73% and has lent funds for 6 months at 7.87%. The total amount is USD38 million.

   To cover his exposure created by the mismatch of maturities, the dealer needs to borrow another USD38 million for months, in 3 months' time, and hedge the position now with a FRA.

   The market has the following quotes from three dealers:

BANK A	3×6	6.92–83
BANK B	3×6	6.87–78
BANK C	3×6	6.89–80

   (a) What is (are) the exposure(s) of this treasurer? Represent the result on cash flow diagrams.
   (b) Calculate this treasurer's break-even forward rate of interest, assuming no other costs.
   (c) What is the best FRA rate offered to this treasurer?
   (d) Calculate the settlement amount that would be received (paid) by the treasurer if, on the settlement date, the Libor fixing was 6.09%.

3. A corporation will receive USD7 million in 3 months' time for a period of 3 months. The current 3-month interest rate quotes are 5.67 to 5.61. The Eurodollar futures price is 94.90.

   Suppose in 3 months the interest rate becomes 5.25% for 3-month Eurodeposits and the Eurodollar futures price is 94.56.

   (a) How many ticks has the futures price moved?
   (b) How many futures contracts should this investor buy or sell if she wants to lock in the current rates?
   (c) What is the profit (loss) for an investor who bought the contract at 94.90?

4. Suppose practitioners learn that the British Banker's Association (BBA) will change the panel of banks used to calculate the yen Libor. One or more of the "weaker" banks will be replaced by "stronger" banks at a future date.

   The issue here is not whether yen Libor will go down, as a result of the panel now being "stronger." In fact, due to market movements, even with stronger banks in the panel, the yen Libor may in the end go up significantly. Rather, what is being anticipated is that the yen Libor should decrease in London *relative* to other yen fixings, such as Tibor. Thus, to benefit from such a BBA move, the market practitioner must form a position where the risks originating from market movements are eliminated and the "only" relevant variable remains the decision by the BBA.

(a) How would a trader benefit from such a change without taking on too much risk?
(b) Using cash flow diagrams, show how this can be done.
(c) In fact, show which *spread* FRA position can be taken. Make sure that the position is (mostly) neutral toward market movements and can be created, the only significant variable being the decision by the BBA.

(From IFR, issue 1267) Traders lost money last week following the British Bankers' Association (BBA) decision to remove one Japanese bank net from the yen Libor *fixing panel. The market had been pricing in no significant changes to the panel just the day before the changes were announced.*

Prior to the review, a number of dealers were reported to have been short the Libor/Tibor spread by around 17 bp, through a twos into fives forward rate agreement (FRA) spread contract. This was in essence a bet that the Japanese presence on the Libor *fixing panel would be maintained.*

When the results of the review were announced on Wednesday January 20, the spread moved out by around 5 bp to around 22 bp—leaving the dealers with mark-to-market losses. Some were also caught out by a sharp movement in the one-year yen/dollar Libor *basis swap, which moved in from minus 26 bp to minus 14 bp.*

The problems for the dealers were caused by BBA's decision to alter the nature of the fixing panel, which essentially resulted in one Japanese bank being removed to be replaced by a foreign bank. Bank of China, Citibank, Tokai Bank and Sakura were taken out, while Deutsche Bank, Norinchukin Bank, Rabobank and WestLB were added.

The move immediately increased the overall credit quality of the grouping of banks responsible for the fixing rate. This caused the yen Libor *fix—the average cost of panel banks raising funds in the yen money market—to fall by 8 bp in a single day. Dealers said that one Japanese bank was equivalent to a 5 bp lower yen* Libor *rate and that the removal of the Bank of China was equivalent to a 1 bp or 2 bp reduction.*

Away from the immediate trading losses, market reaction to the panel change was mixed. The move was welcomed by some, who claimed that the previous panel was unrepresentative of the yen cash business being done.

"Most of the cash is traded in London by foreign banks. It doesn't make sense to have half Japanese banks on the panel," said one yen swaps dealer. He added that because of the presence of a number of Japanese banks on the panel, yen Libor *rates were being pushed above where most of the active yen cash participants could fund themselves in the market.*

Others, however, disagreed. "It's a domestic [Japanese] market at the end of the day. The BBA could now lose credibility in Japan," said one US bank money markets trader.

BBA officials said the selections were made by the BBA's FX and Money Markets Advisory Panel, following private nominations and discussions with *the BBA* Libor *Steering Group. They said the aim of the advisory panel was to ensure that the contributor panels broadly reflected the "balance of activity in the interbank deposit market."*

5. You are hired by a financial company in New Zealand and you have instant access to markets. You would like to lock in a 3-month borrowing cost in NZ$ for your client. You consider a NZ$ 1×4 FRA. But you find that it is overpriced as the market is thin.

   So you turn to Aussie. A$ FRAs are very liquid. It turns out that the A$ and NZ$ forwards are also easily available.

   In particular, you obtain the following data from Reuters:

A$/NZ$	Spot the:	1.17/18
	1-m forward:	1.18/22
	3-m forward:	1.19/23
	4-m forward:	1.28/32
A$ FRA's	1×4	8.97

   (a) Show how you can create a 1×4 NZ$ from these data.
   (b) Show the cash flows.
   (c) What are the risks of your position (if any) compared to a direct 1×4 NZ$ FRA?
   (d) To summarize the lessons learned from this exercise (if any), do you think there must be arbitrage relationships between the FRA markets and currency forwards? Explain. Or better, provide the relevant formulas.

6. You are given the following information:

3-m Libor	3.2%	92 days
3×6 FRA	3.3%–3.4%	90 days
6×9 FRA	3.6%–3.7%	90 days
9×12 FRA	3.8%–3.9%	90 days

   (a) Show how to construct a synthetic 9-month loan with fixed rate beginning with a 3-month loan. Plot the cash flow diagram.
   (b) What is the fixed 9-month borrowing cost?